PROGETTO REFRESCOS :
WP5

Simulazione numerica del fenomeno di distacco tra materiali eterogenei soggetti a variazioni di temperatura mediante il modello della fessura coesiva all’ambito termoelastico

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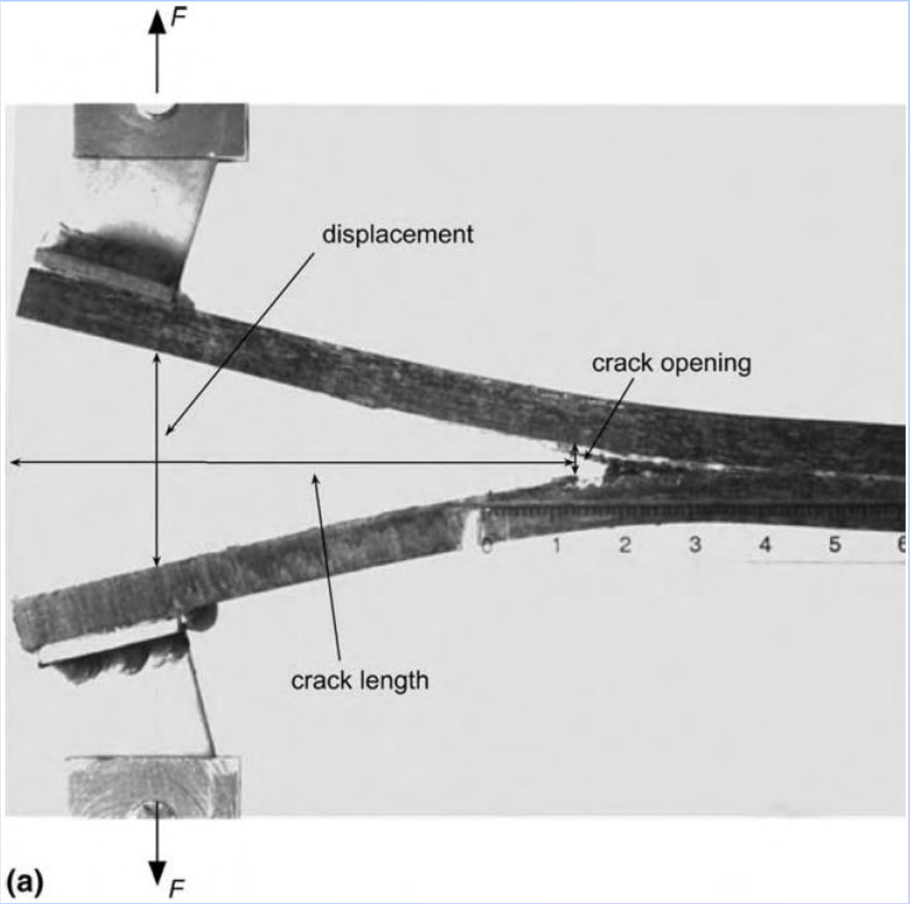
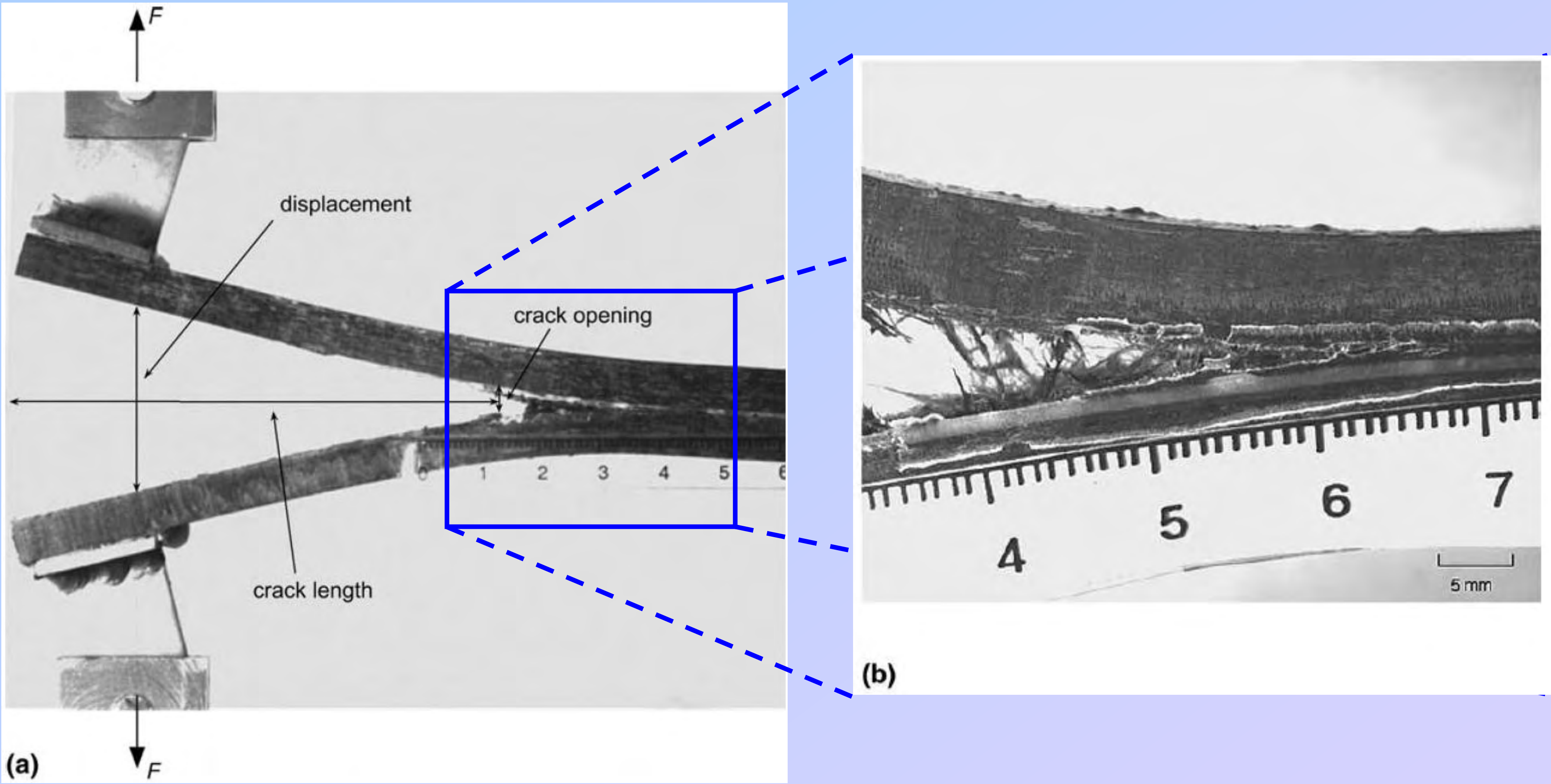
Outline

- **Introduction on the cohesive zone model**
- **Extension of the classical cohesive zone model to coupled thermoelastic problems**
- **Preliminary example showing a thermally induced crack growth at a bi-material interface**

Interface fracture – the CZM



Interface fracture – the CZM



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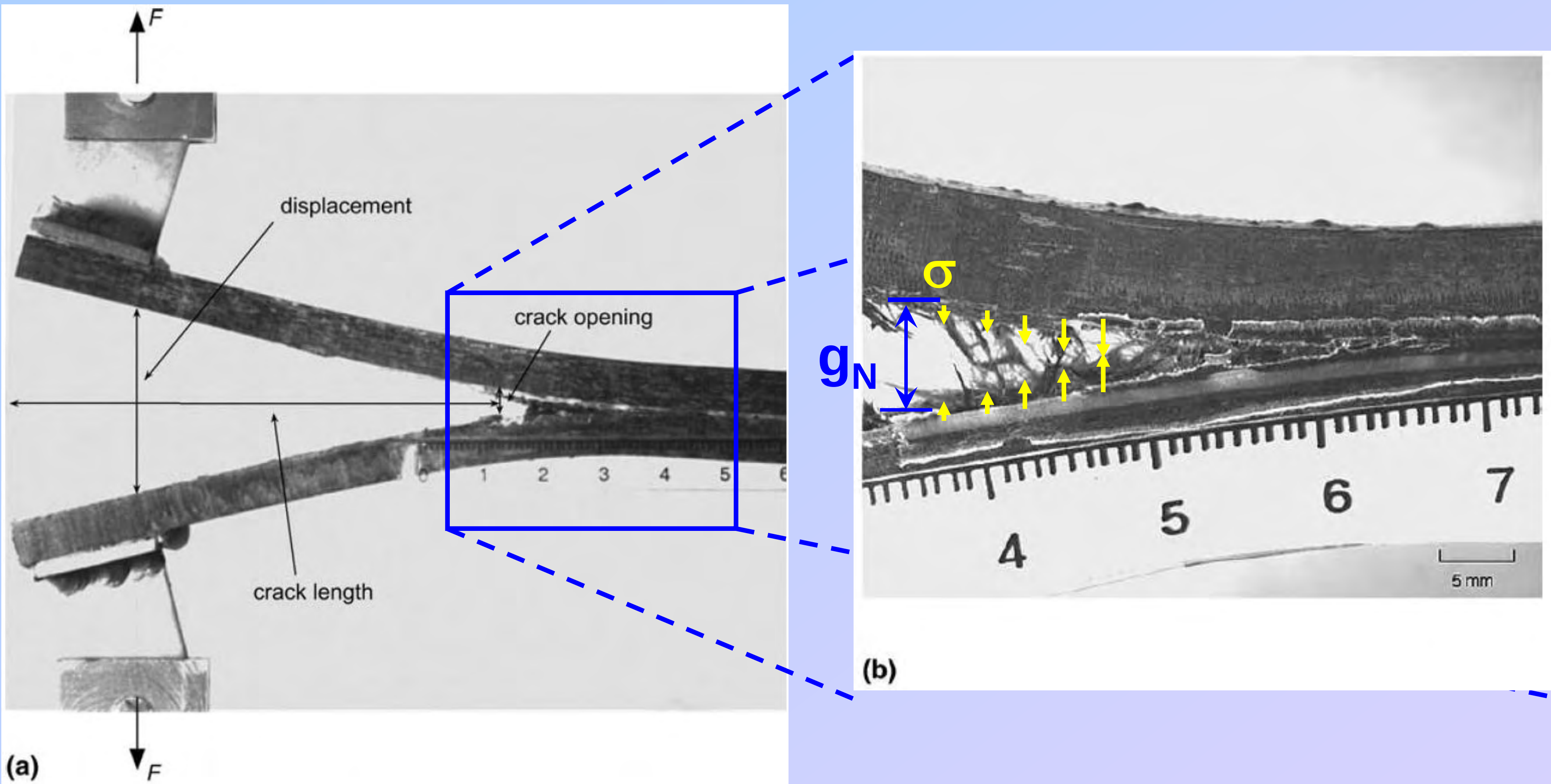
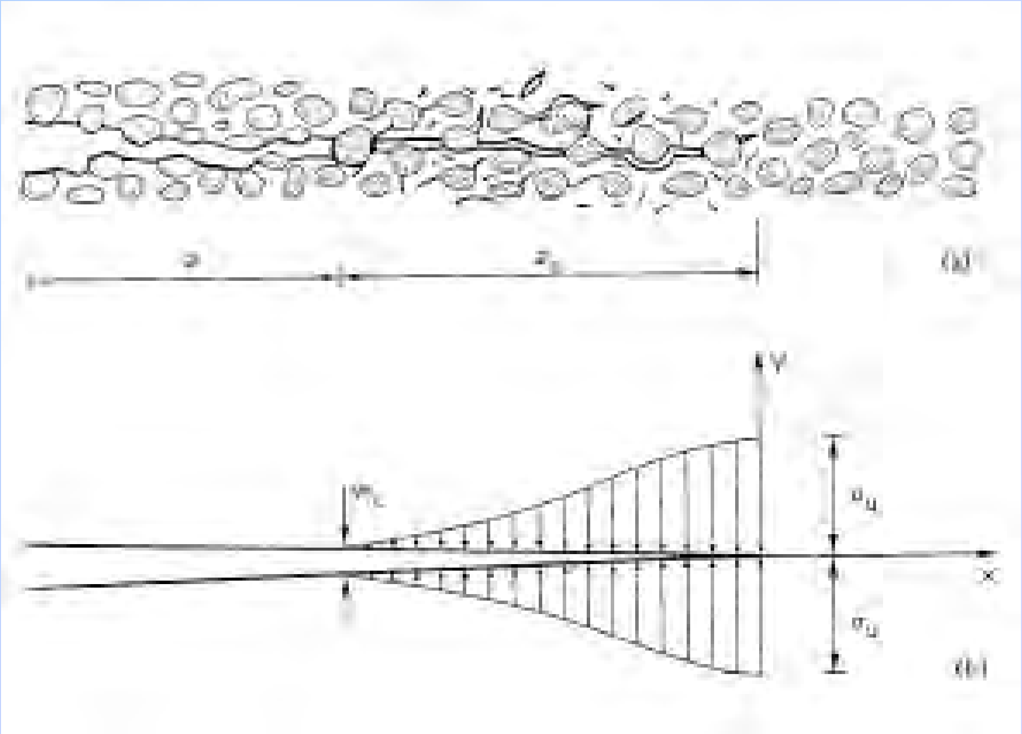


Cohesive cracking in quasi-brittle materials



Carpinteri, *J Mech Phys Solids* (1989)
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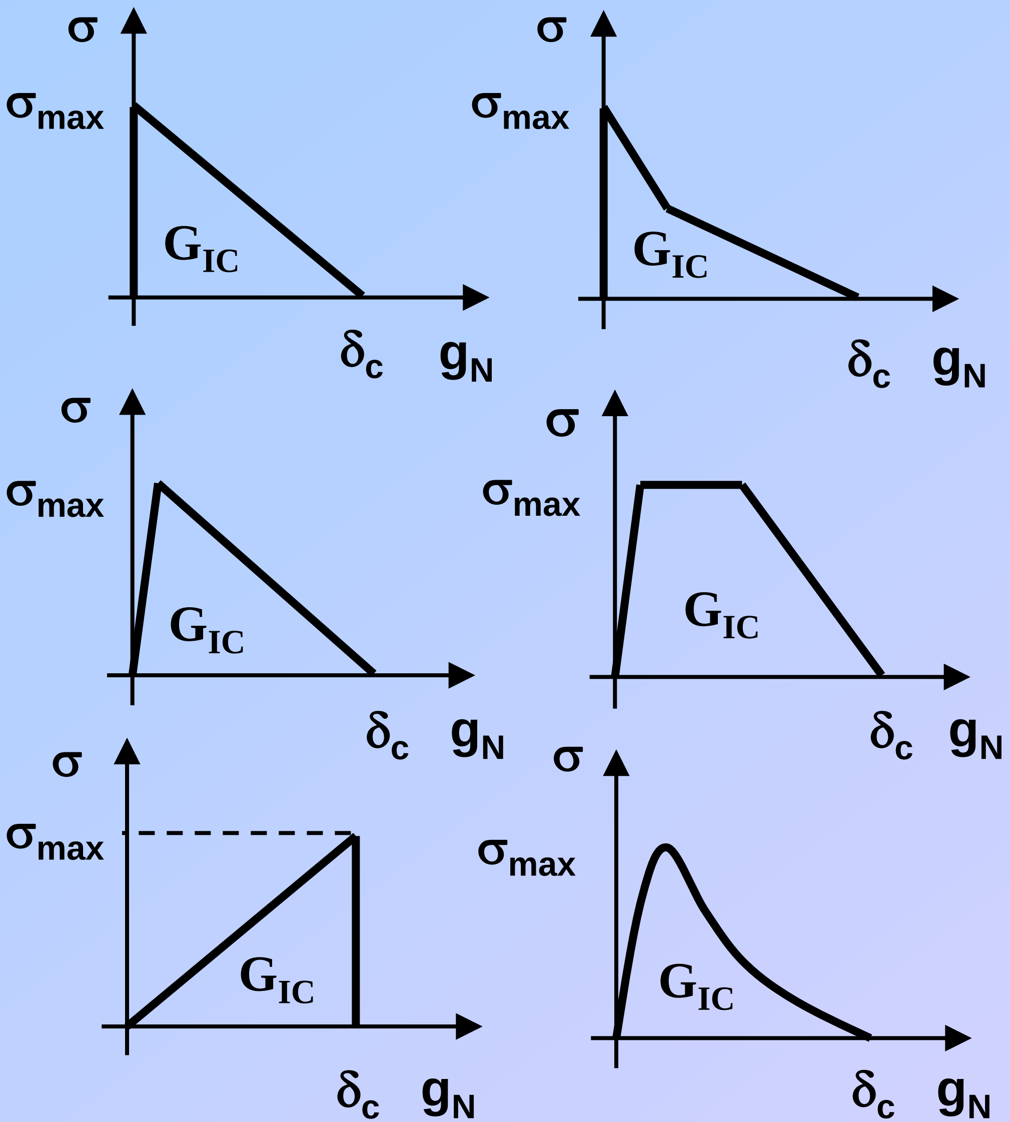
Cohesive Zone Models (CZMs)



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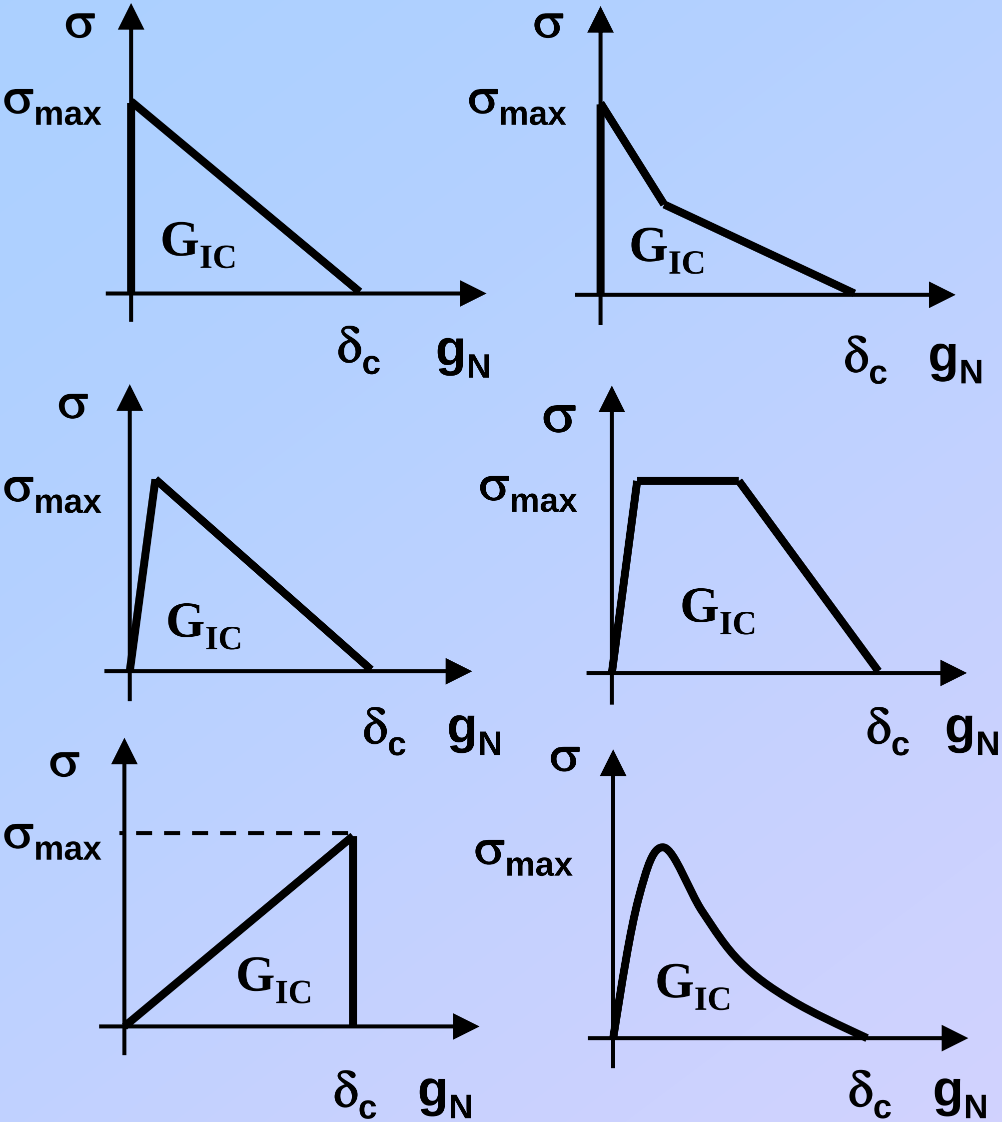
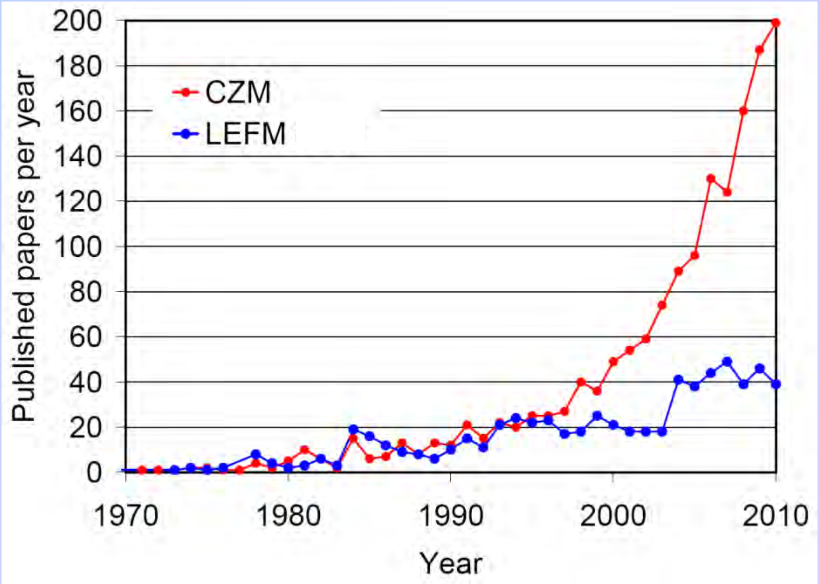
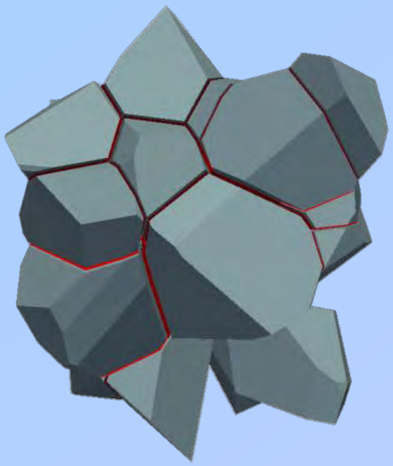


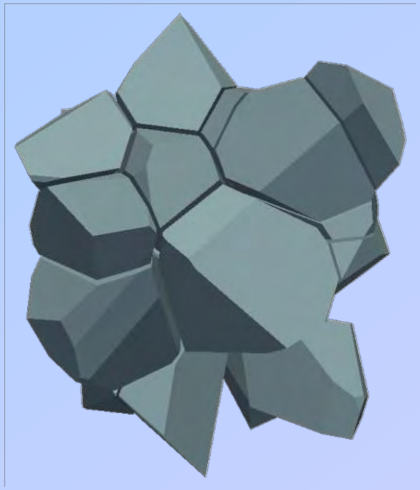


Source: SCOPUS

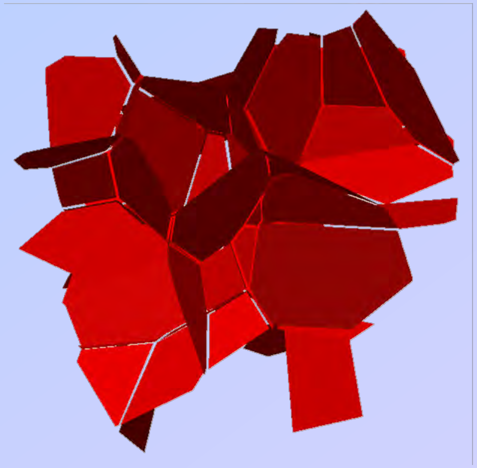
Application to intergranular fracture



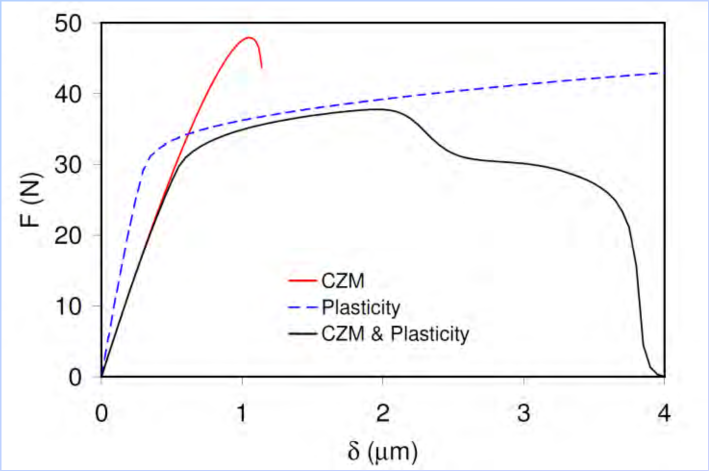
3D polycrystal

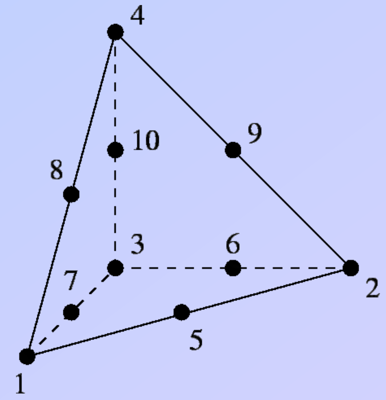


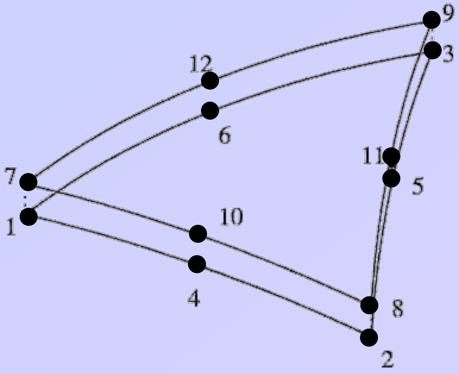
Grains



Grain boundaries







Paggi, Lehmann, Weber, Carpinteri, Wriggers (2012)

Analysis of debonding phenomena in decorated mural elements by numerical models based on Fracture Mechanics:

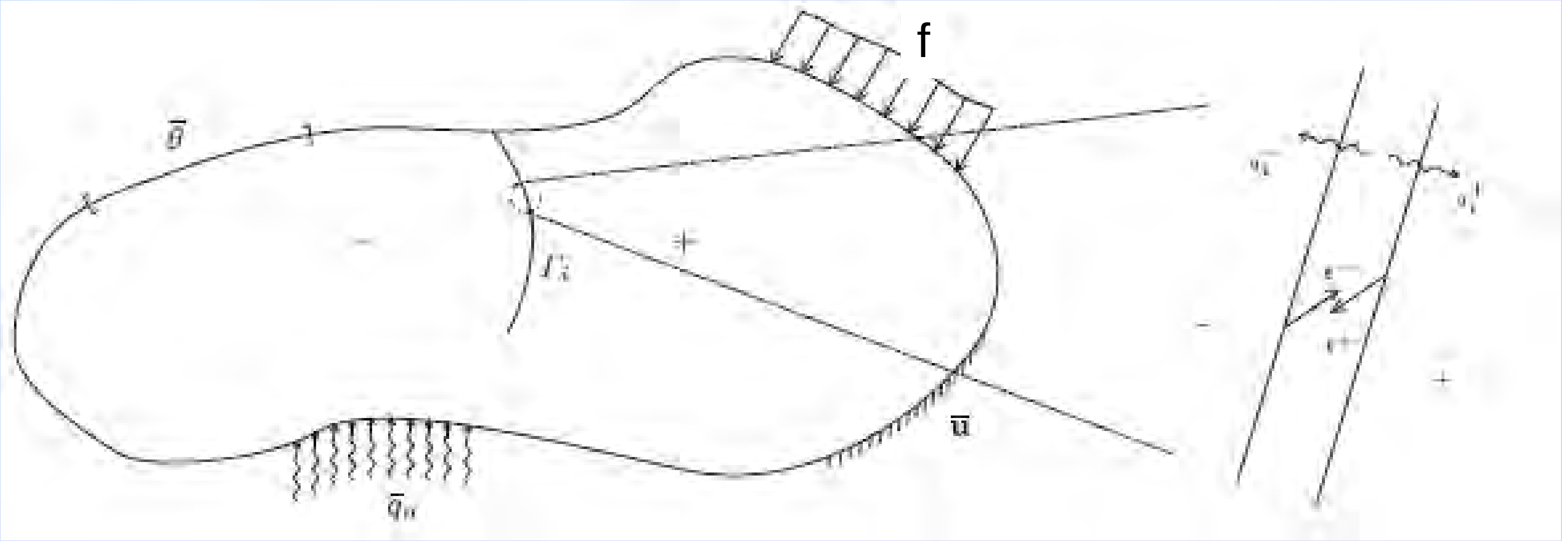
- **Thermal effects**
- **Diffusion problems (permeability, moisture)**

Thermoelasticity: Weak form of the governing eqs

$$\int_V (\nabla\delta\mathbf{u})^{\mathrm{T}}\boldsymbol{\sigma}\,\mathrm{d}V = \int_{\partial V}\delta\mathbf{u}^{\mathrm{T}}\mathbf{f}\,\mathrm{d}S + \int_S \delta\mathbf{g}^{\mathrm{T}}\mathbf{t}\,\mathrm{d}S$$

$$\int_V q\;\nabla\delta T\,\mathrm{d}V = \int_V \rho c_V \dot{T}\,\delta T\,\mathrm{d}V + \int_{\partial V} q_n\,\delta T\,\mathrm{d}S + \int_S q_s\,\delta T\,\mathrm{d}S$$

CZM contributions



FE implementation of the thermoelastic CZM

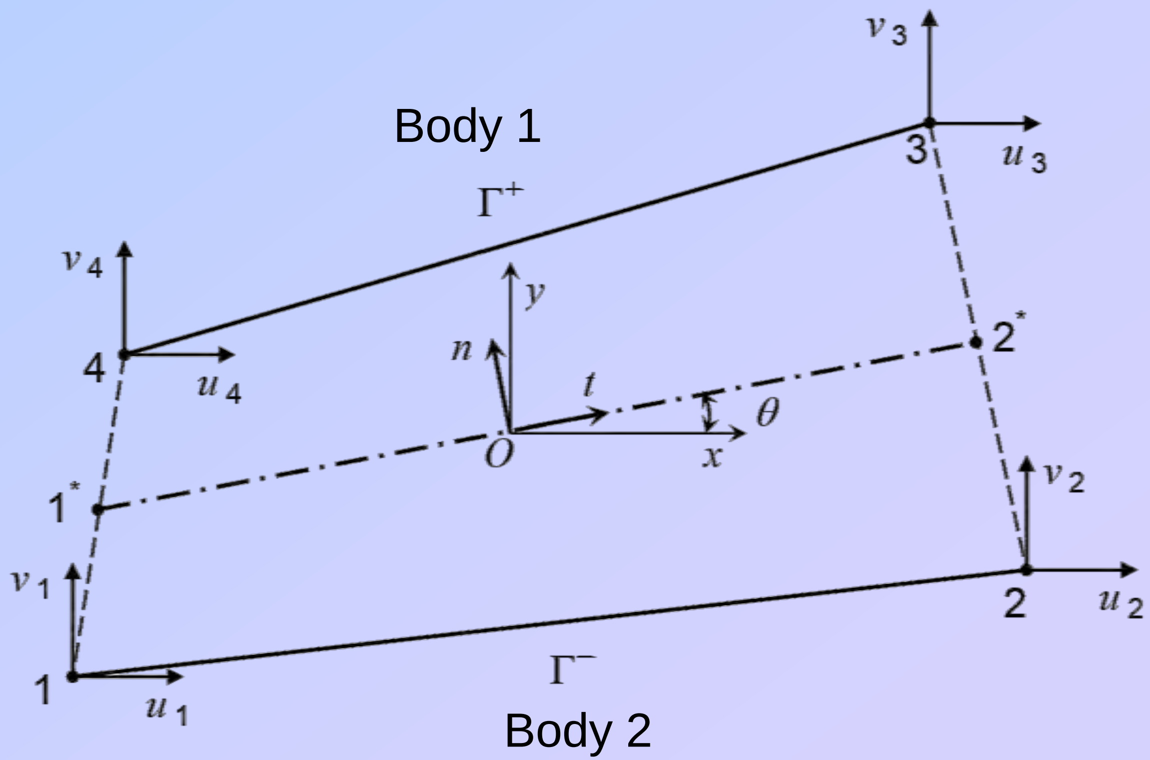
Nodal displacements and temperatures:

$$\mathbf{u} = (u_1, v_1, T_1, u_2, v_2, T_2, u_3, v_3, T_3, u_4, v_4, T_4)^{\mathrm{T}}$$

$$\mathbf{u}^* = \mathbf{R}\mathbf{u}$$

$$\mathbf{R} = \begin{bmatrix} \mathbf{r} & & & \\ & \mathbf{r} & & \\ & & \mathbf{r} & \\ & & & \mathbf{r} \end{bmatrix}$$

$$\mathbf{r} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



FE implementation of the thermoelastic CZM

<u>Relative</u> nodal displacements and temperatures:

$$\Delta\mathbf{u}^{*}=(u_4^{*}-u_1^{*},v_4^{*}-v_1^{*},T_4-T_1,u_3^{*}-u_2^{*},v_3^{*}-v_2^{*},T_3-T_2)^{\mathrm{T}}$$

$$\Delta\mathbf{u}^{*}=\mathbf{L}\mathbf{u}^{*}$$

$$\mathbf{L}=\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 \\ 0 & 0 & 0 & -1 & 0 & 0 & +1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & +1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & +1 & 0 & 0 & 0 \end{bmatrix}$$

FE implementation of the thermoelastic CZM

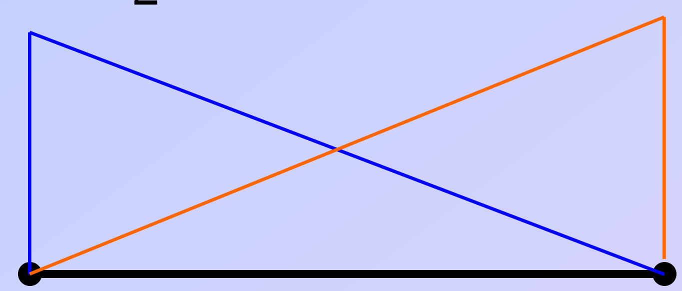
Relative displacements and temperatures in a generic point:

$$\mathbf{g} = (g_{\mathrm{T}}, g_{\mathrm{N}}, \Delta T)^{\mathrm{T}}$$

$$\mathbf{g} = \mathbf{N}\Delta\mathbf{u}^{*}$$

$$\mathbf{N} = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 \end{bmatrix}$$

$$N_1 = \frac{1-s}{2} \qquad N_2 = \frac{1-s}{2}$$



FE implementation of the thermoelastic CZM

Interface contribution to the weak form:

$$G_{\mathrm{int}} = \int_S \delta\mathbf{g}^{\mathrm{T}}\mathbf{p}\,\mathrm{d}S = \int_S \left(\delta g_{\mathrm{T}}, \delta g_{\mathrm{N}}, \delta\Delta T\right)\begin{pmatrix}\tau \\ \sigma \\ q_{\mathrm{S}}\end{pmatrix}\mathrm{d}S$$

The nonlinear dependency between the vector *p* and the gap vector *g* has to be linearized for the application of the Newton-Raphson method (Paggi and Wriggers, 2011):

$$\Delta G_{\mathrm{int}} = \int_S \left(\delta g_{\mathrm{T}}, \delta g_{\mathrm{N}}, \delta\Delta T\right)\mathbf{C}\begin{pmatrix} g_{\mathrm{T}} \\ g_{\mathrm{N}} \\ \Delta T\end{pmatrix}\mathrm{d}S$$

FE implementation of the thermoelastic CZM

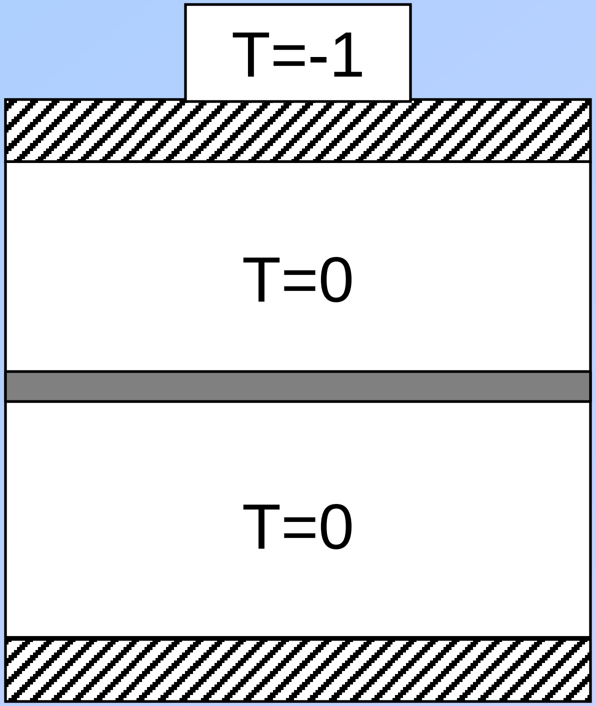
Tangent interface constitutive matrix C:

$$\mathbf{C}=\left[\begin{array}{c:c}\mathbf{C}_{\mathrm{M}} & 0 \\ \hdashline \mathbf{C}_{\mathrm{TM}} & C_{\mathrm{T}}\end{array}\right]=\left[\begin{array}{cc:c}\dfrac{\partial\tau}{\partial g_{\mathrm{T}}} & \dfrac{\partial\tau}{\partial g_{\mathrm{N}}} & 0 \\ \dfrac{\partial\sigma}{\partial g_{\mathrm{T}}} & \dfrac{\partial\sigma}{\partial g_{\mathrm{N}}} & 0 \\ \hdashline 0 & \dfrac{\partial q_{\mathrm{S}}}{\partial g_{\mathrm{N}}} & \dfrac{\partial q_{\mathrm{S}}}{\partial \Delta T}\end{array}\right]$$

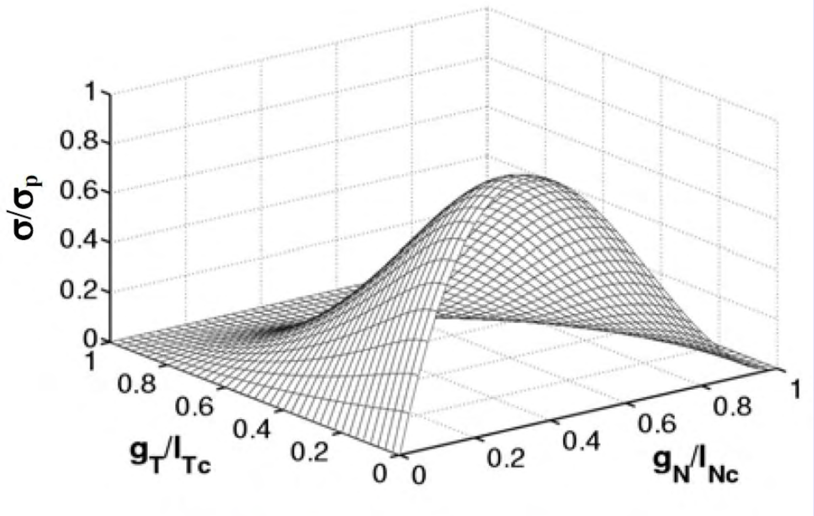
where thermo-mechanical coupling is related to the nonlinear dependency between heat flux and crack opening displacement (Paggi and Barber, 2011):

$$q_{\mathrm{S}}=-k_{\mathrm{int}}(g_{\mathrm{N}},\Delta T)\,\Delta T$$

Example

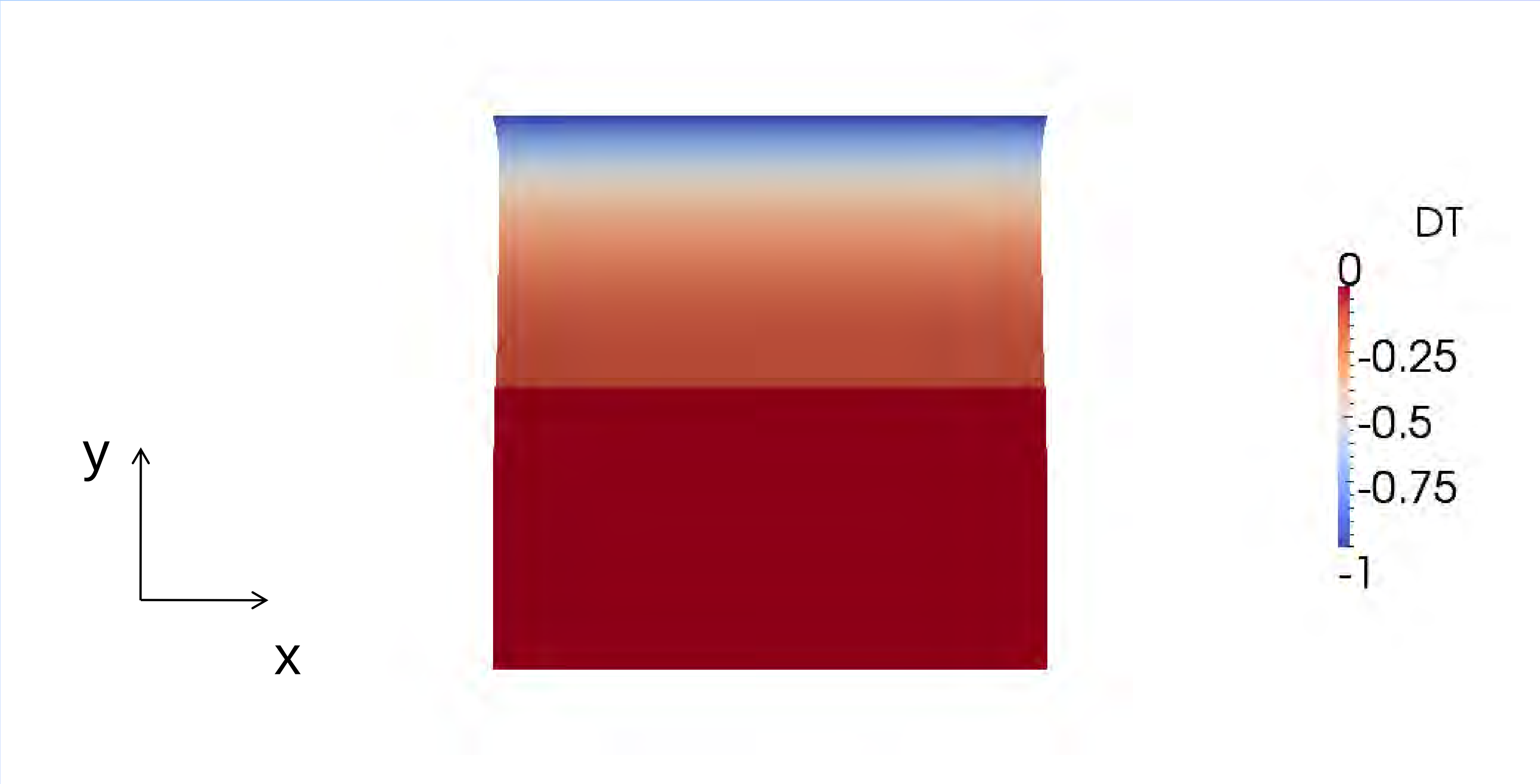


k_{int}: *constant*



(a) Normal cohesive tractions

Example: thermally-induced debonding



Conclusions

A cohesive zone model has been proposed for the analysis of debonding phenomena at bimaterial interfaces. Future developments include:

- **Thermal effects - different materials by properly choosing *ad-hoc* parameters (plaster/masonry)**
- **To investigate diffusion problems (permeability) in order to study the moisture effects/percolation**